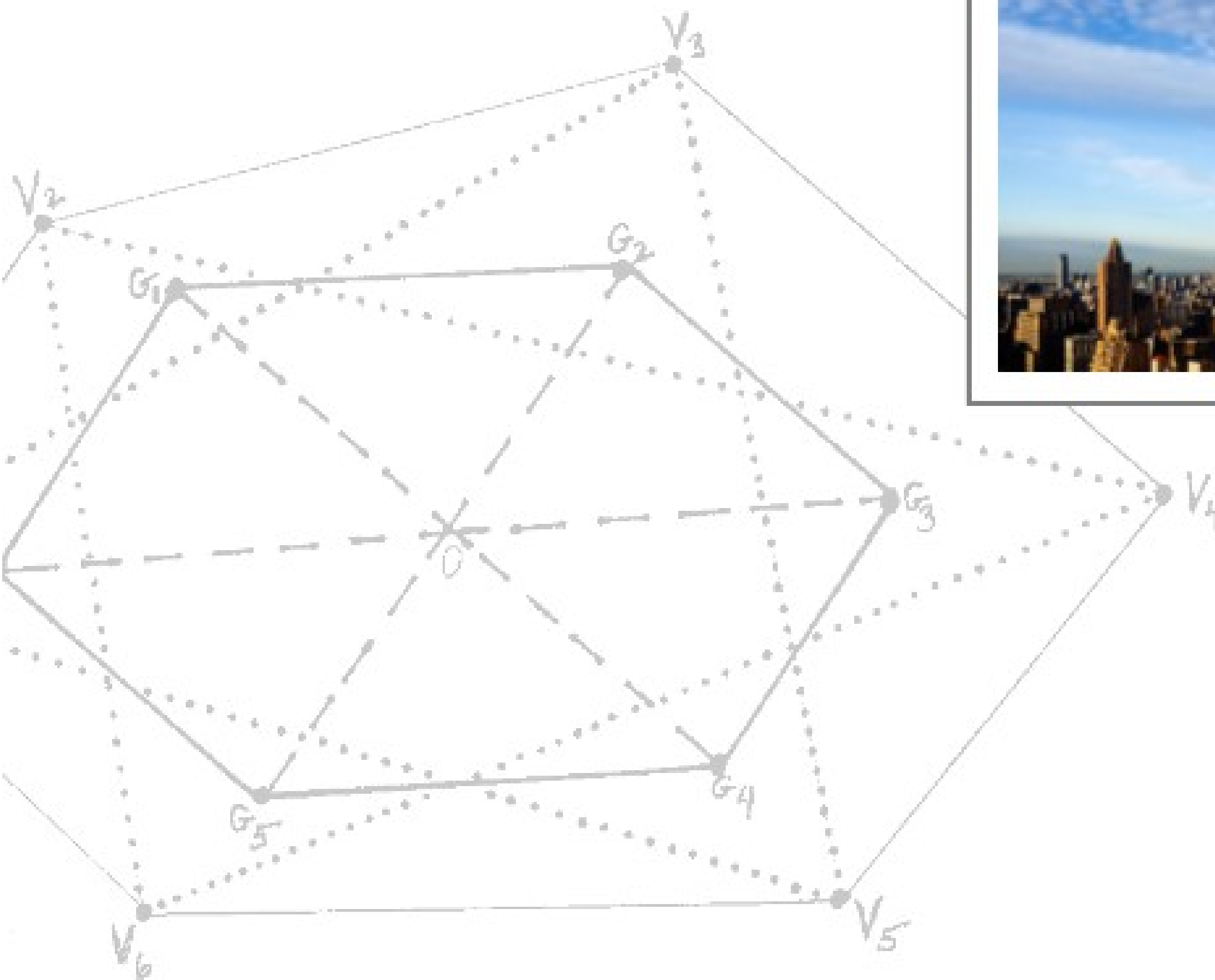
# Straight Line






NOTE

## Gradient of a Straight Line

The symbol $\triangle$ is called 'delta' and is used to describe change.

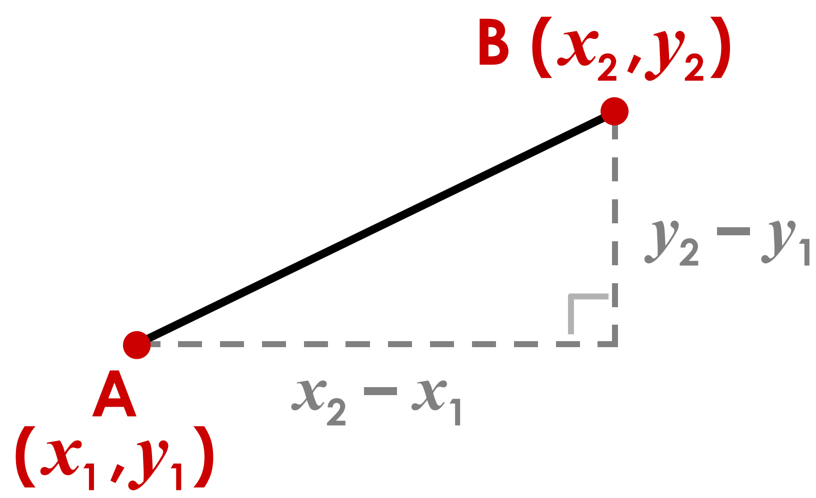
### Example

(-2,7) (3,9)

$$\triangle x = 3 - (-2)$$
$$= 5$$

### Gradient

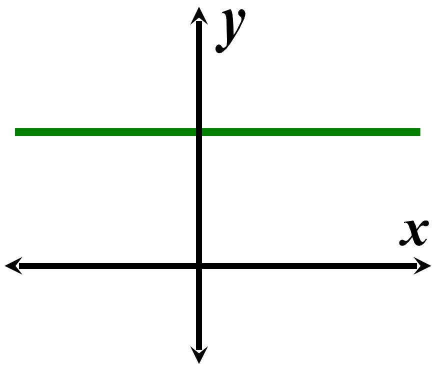
$$m = \frac{\triangle y}{\triangle x} = \frac{y_2 - y_1}{x_2 - x_1}$$



NOTE

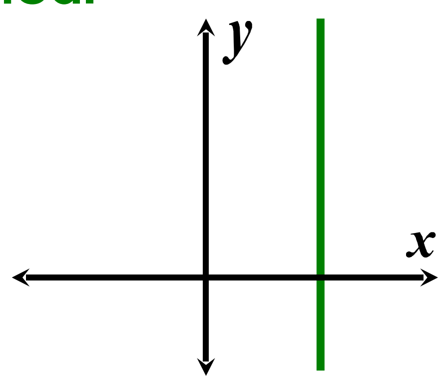
# Horizontal and Vertical Lines

## Horizontal




- zero gradient
- parallel to $x$-axis
- equations written in the form $y = \square$

## Vertical

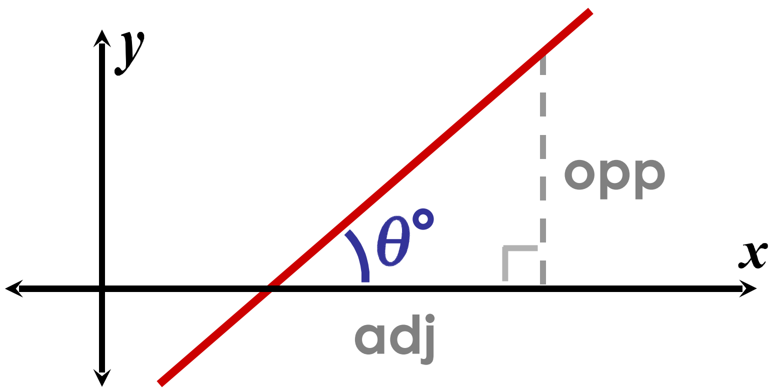



- infinite gradient ('undefined')
- parallel to $y$-axis
- equations written in the form $x = \square$

NOTE

## Gradient and Angle

The symbol $\theta$ is called '**theta**' and is often used for angles.



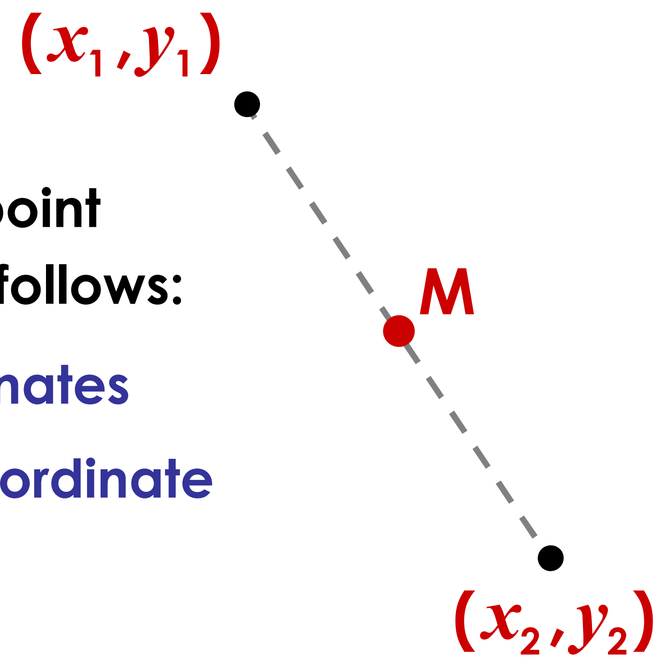

$$\tan\theta^\circ = \frac{\text{opp}}{\text{adj}} = \frac{\Delta y}{\Delta x} = m$$

$$m = \tan\theta^\circ$$

where $\theta^\circ$ is the angle between the line and the **positive** direction of the $x$-axis.

NOTE

## Finding the Midpoint




The midpoint M of two coordinates is the point halfway in between and can be found as follows:

- find the distance between the $x$-coordinates
- add half of this distance to the first $x$-coordinate
- do the same again for the $y$-coordinate
- write down the midpoint

An alternative method is to find the average of both coordinates:

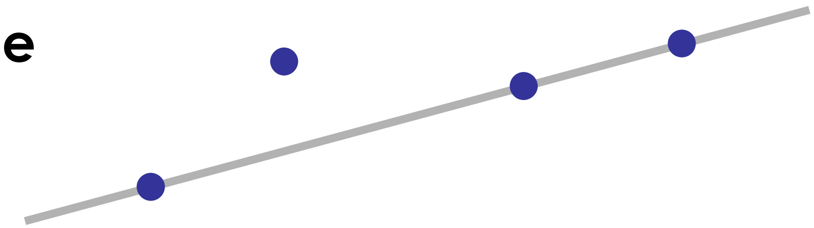
$$M = \left( \frac{x_2 + x_1}{2}, \frac{y_2 + y_1}{2} \right)$$

The Midpoint Formula

NOTE

## Collinearity

If points lie on the same straight line they are said to be **collinear**.



### Example

Prove that the points P(-6,-5), Q(0,-3) and R(12,1) are collinear.

$$m_{PQ} = \frac{(-3) - (-5)}{0 - (-6)} = \frac{2}{6} = \frac{1}{3}$$

$$m_{QR} = \frac{1 - (-3)}{12 - 0} = \frac{4}{12} = \frac{1}{3}$$

We have shown

$$m_{PQ} = m_{QR}$$

$\therefore$ The points are collinear.

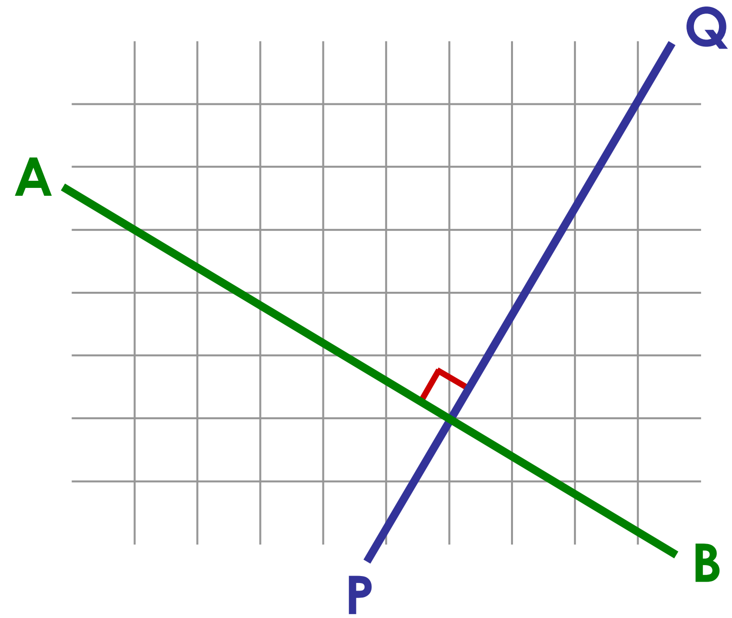
NOTE

## Gradients of Perpendicular Lines

When two lines are at 90° to each other we say they are perpendicular.

If the lines with gradients $m_1$ and $m_2$ are perpendicular,

$$m_1 = \frac{-1}{m_2}$$




$m_{AB} = \frac{-3}{5}$ $m_{PQ} = \frac{5}{3}$

To find the gradient of a perpendicular line:

- flip the fraction upside down
- change from positive to negative (or negative to positive)

NOTE

## Different Types of Straight Line Equation

Any straight line can be described by an equation.

The equation of a straight line can be written in several different ways .

**Basic Equation** (useful for sketching)

$$y = mx + c$$

**General Equation** (always equal to zero)

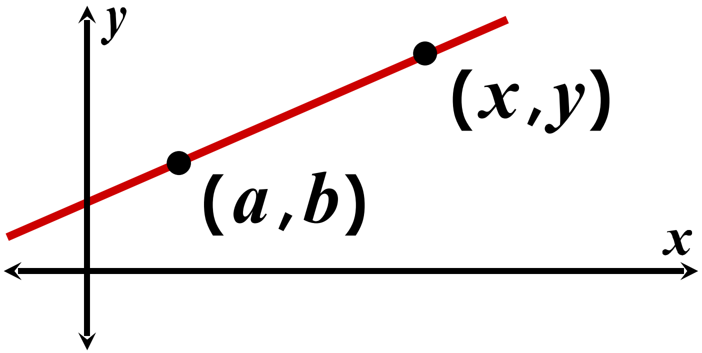
$$Ax + By + C = 0$$

(not the $y$-intercept!)

Straight lines are normally described using the General Equation, with $A$ as a positive number.

NOTE

## The Alternative Straight Line Equation




$$m = \frac{y - b}{x - a}$$

$$\therefore \quad y - b = m(x - a)$$

for the line with gradient $m$ passing through $(a, b)$

## Example

Find the equation of the line with gradient $\frac{1}{2}$ and passing through (5,-2).

**Substitute:**

$$y - (-2) = \tfrac{1}{2}(x - 5)$$

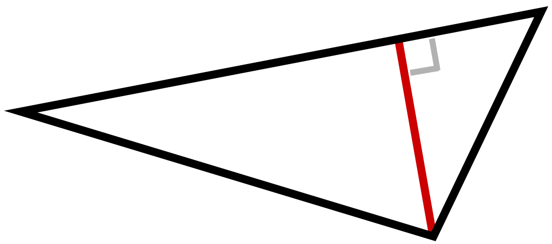
$$y + 2 = \tfrac{1}{2}(x - 5)$$

$$2y + 4 = x - 5$$

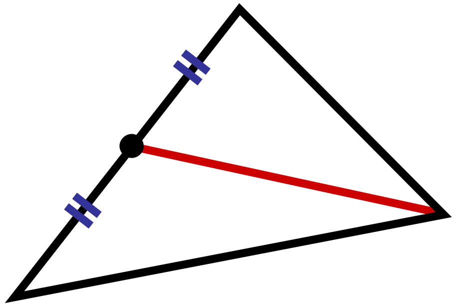
$$x - 2y - 9 = 0$$

NOTE

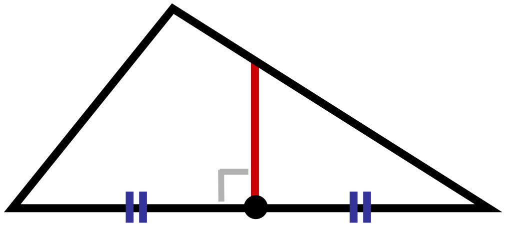
## Straight Lines in Triangles



An **altitude** is a perpendicular line between a vertex and the opposite side.



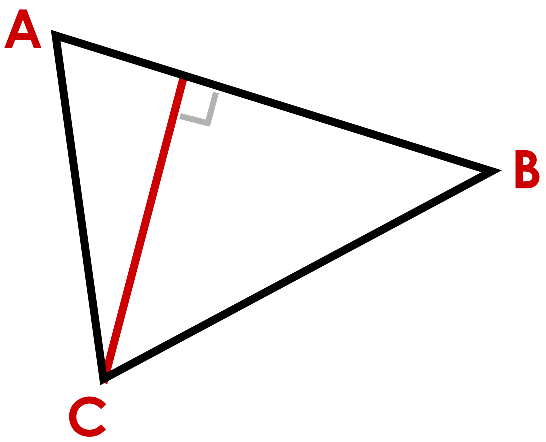
A **median** is a line between a vertex and the midpoint of the opposite side.



A **perpendicular bisector** is a perpendicular line from the midpoint of a side.

NOTE

## Finding the Equation of an Altitude




To find the equation of an altitude:

- Find the gradient of the side it is perpendicular to ( $m_{AB}$ ).
- To find the gradient of the altitude, flip the gradient of AB and change from positive to negative: $m_{altitude} = \frac{-1}{m_{AB}}$
- Substitute the gradient and the point C into

$$y - b = m(x - a)$$

**Important**

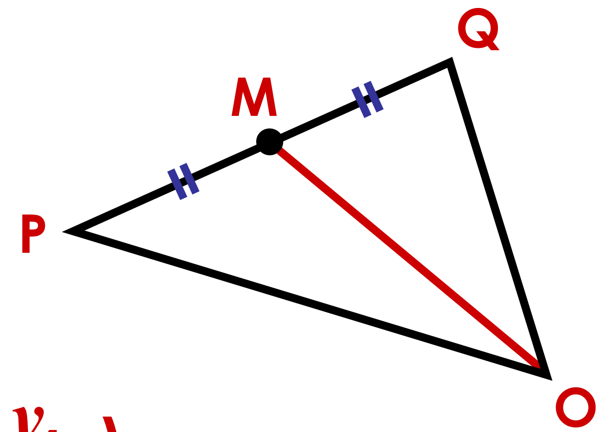
Write final equation in the form

$$Ax + By + C = 0$$

with $Ax$ positive.

NOTE

## Finding the Equation of a Median




To find the equation of a median:

- Find the midpoint of the side it bisects, i.e.

$$M = \left( \frac{x_2 + x_1}{2}, \frac{y_2 + y_1}{2} \right)$$

- Calculate the gradient of the median OM.

- Substitute the gradient and either point on the line (O or M) into

$$y - b = m(x - a)$$

**Important**

Write answer in the form

$$Ax + By + C = 0$$

with $Ax$ positive.