

The rate of increase of bacteria, b , in a petri dish is directly proportional to the amount of bacteria present at time t .

$$\frac{db}{dt} = kb$$

Form the differential equation.

$$\int \frac{db}{b} = \int k dt$$

We will separate the variables, and split the differential. When we do this we will introduce an integration sign.

$$\ln b + c_1 = kt + c_2$$

Integrate both sides. Note on the right hand side we are integrating with respect to t (k is constant); secondly there is a constant on both sides that we have combined to give C . This is the **general solution**.

$$\ln b = kt + C$$

The rate of increase of bacteria, b , in a petri dish is directly proportional to the amount of bacteria present at time t . Initially there are 25 mg of bacteria and after 10 hours there are 55 mg of bacteria. Find an equation for b in terms of t .

$$\ln b = kt + C$$

$$b = e^{kt+C}$$

$$b = e^{kt} e^C$$

$$b = Ae^{kt}$$

The general solution rearranged, so b is the subject of the equation. Note e^C , is now the constant number A .

$$25 = Ae^{0k} \rightarrow A = 25$$

Using the initial value $t=0$.

$$55 = 25e^{10k}$$

$$e^{10k} = \frac{55}{25}$$

$$10k = \ln\left(\frac{55}{25}\right)$$

$$k = 0.0788$$

Using the 2nd value, $t=10$ and $b=55$.

So we have the **particular solution**:

$$b = 25e^{0.0788t}$$