

DP Analysis & Approaches (SL)

Maths with Lemon

Lesson 2.8 Function Inverses

Mathematician: _____

SL 2.2	Guidance, clarification and syllabus links
Informal concept that an inverse function reverses or undoes the effect of a function. Inverse function as a reflection in the line $y = x$, and the notation $f^{-1}(x)$.	Solving $f(x) = 10$ is equivalent to finding $f^{-1}(10)$. Inverse functions exist for one-to-one functions; the domain of f^{-1} equals the range of f .
SL 2.5	Guidance, clarification and syllabus links
Identity function. Finding $f^{-1}(x)$.	$(f \circ f^{-1})(x) = (f^{-1} \circ f)(x) = x.$ Existence of an inverse for one-to-one functions.

Warm Up

Let

$$f(x) = 3x^2 - 2, \quad g(x) = -2x + 1.$$

Find:

$$(f + g)(x)$$

$$(f - g)(x)$$

$$(f \cdot g)(x)$$

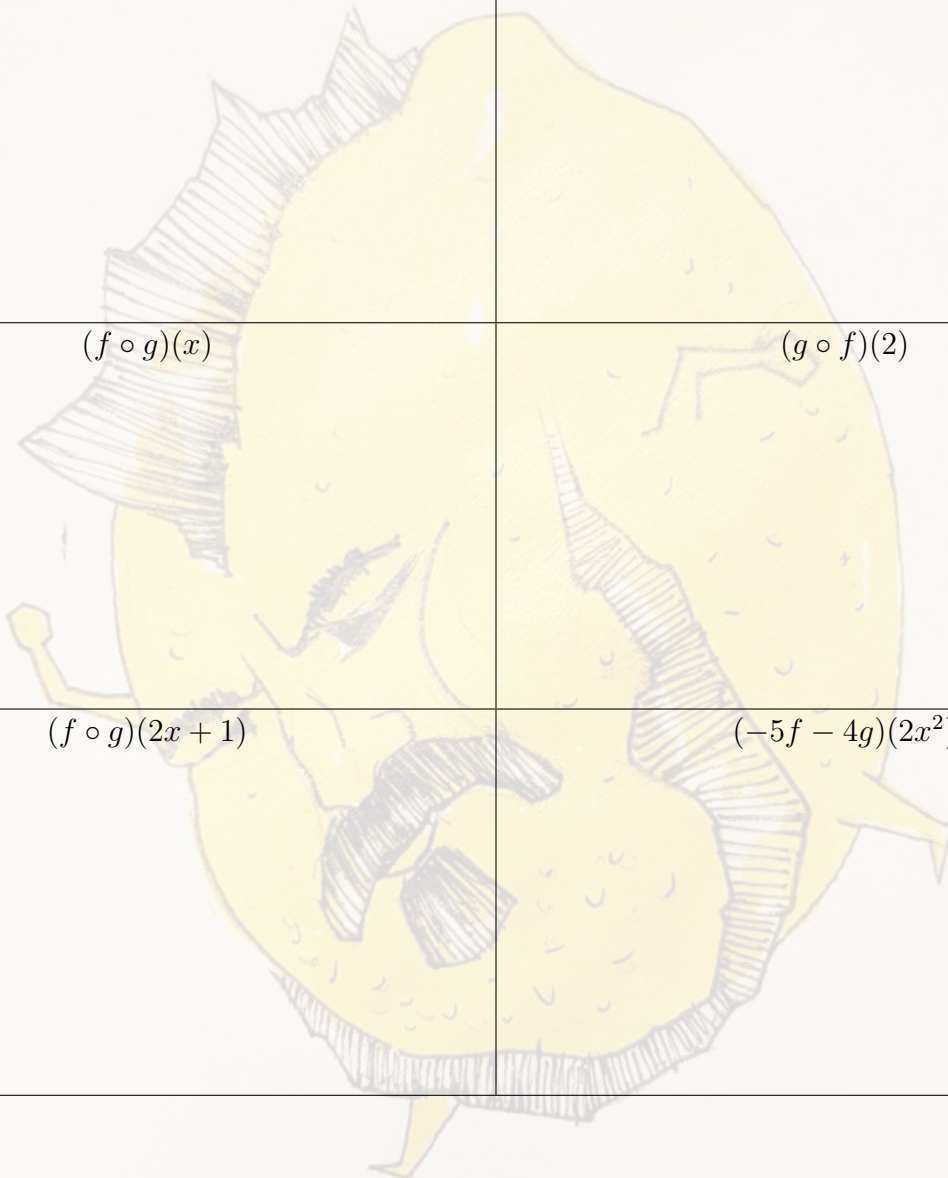
$$(f \div g)(x)$$

$$(f \circ g)(x)$$

$$(g \circ f)(2)$$

$$(f \circ g)(2x + 1)$$

$$(-5f - 4g)(2x^2)$$



Inverse Functions Investigation

Compose the functions below in both directions. What do you notice?

Worked Example	You Try
$f(x) = x + 2, \quad g(x) = x - 2$	$f(x) = 3x, \quad g(x) = \frac{x}{3}$

Showing Two Functions are Inverses

Two functions are inverses if

$$(f \circ g)(x) = (g \circ f)(x) = x.$$

Worked Example	You Try
$f(x) = x^2, \quad g(x) = \sqrt{x}$	$f(x) = 2x - 2, \quad g(x) = \frac{x}{2} + 1$
$f(x) = \frac{-16 + x}{4}, \quad g(x) = 4x + 16$	$f(x) = \frac{1}{x^2 - 1}, \quad g(x) = \sqrt{\frac{1}{x} + 1}$

Matching Inverse Functions

Match each function with its inverse:

$$f(x) = x^2 - 2, \quad g(x) = \sqrt{x} + 2, \quad h(x) = x - 7,$$

$$k(x) = \sqrt{x + 2}, \quad m(x) = x^2 - 4x + 4, \quad p(x) = x + 7.$$

Linear Function Inverses

Find the inverse of each function.

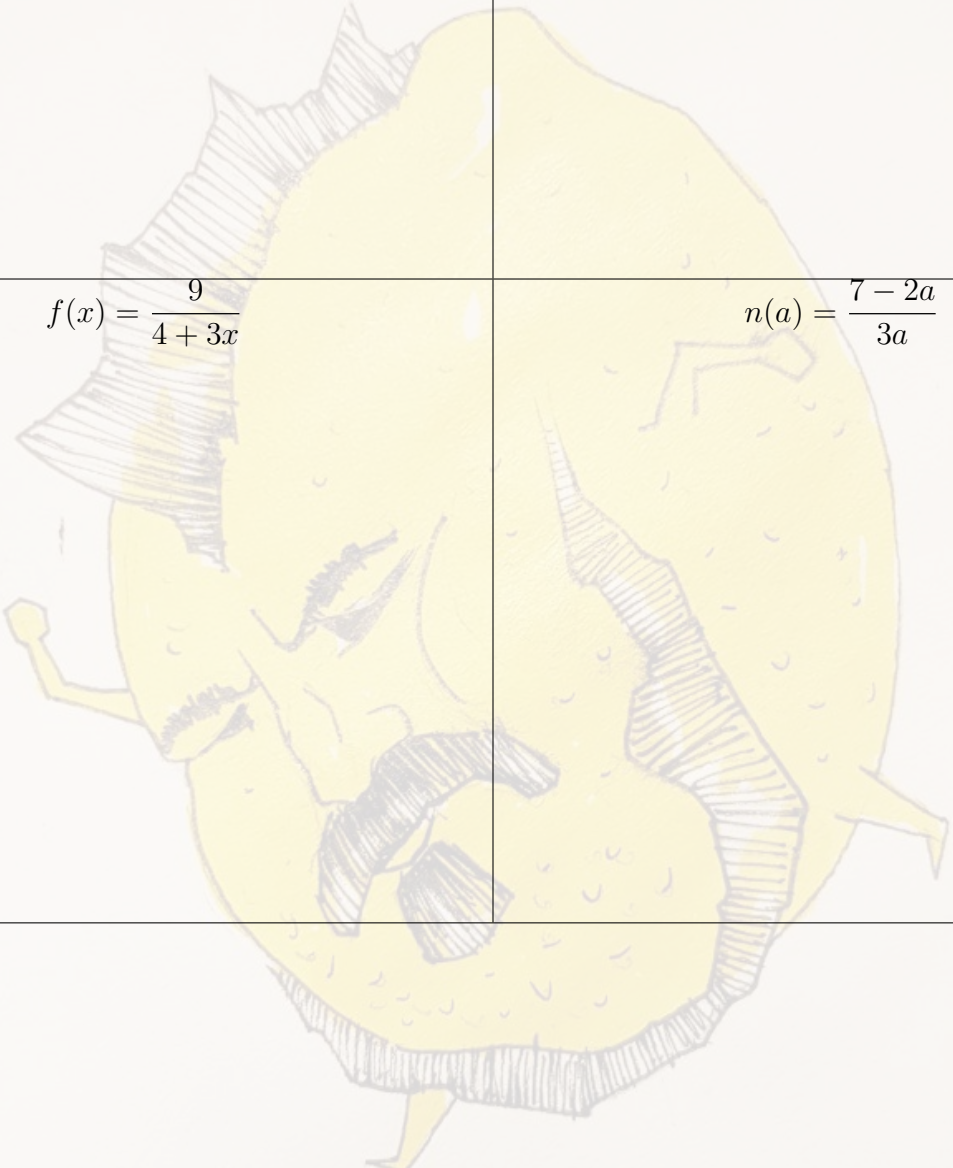
Worked Example	You Try
$f(x) = 2x + 6$	$f(x) = -\frac{4}{5}x - 4$
$f(x) = -\frac{3}{4}x - 3$	$f(x) = \frac{2}{3}x + 12$

Radical Function Inverses

Worked Example	You Try
$h(n) = \sqrt[3]{\frac{n-1}{2}}$	$f(x) = (x^5 + 2)^{1/4}$
$f(x) = \left(\frac{\sqrt[4]{x}}{8}\right)^7$	$g(t) = \frac{t^{1/7} + 9}{4} - 8$

Rational Function Inverses

Worked Example	You Try
$g(x) = \frac{x - 3}{x + 2}$	$g(x) = \frac{x + 3}{2x - 9}$
$f(x) = \frac{9}{4 + 3x}$	$n(a) = \frac{7 - 2a}{3a}$



Sketching Inverses

To sketch an inverse, reverse the x - and y -coordinates of all points, or reflect the graph in

$$\boxed{y = x}.$$

Worked-example points:

$$(-4, -2), (-3, 0), (-1, 1), (0, 3), (4, 4).$$

Inverses from Tables

x	0	1	2	3	4	5	6
$f(x)$	3	6	4	2	0	1	5

Find:

$$\begin{array}{lll} (a) f(1), & (b) f^{-1}(1), & (c) f(3), \\ (d) f^{-1}(3), & (e) f(0), & (f) f^{-1}(0). \end{array}$$

Evaluation of Inverse Functions

Given

$$f(x) = \frac{x}{2} + 5,$$

(a) Evaluate $f(18)$.

(b) Find $f^{-1}(7)$.

Exam Style Questions

Problem 1

Let

$$f(x) = \frac{1}{1 + \sqrt{x}}, \quad g(x) = x + 7.$$

Solve

$$f^{-1}(g^{-1}(x)) = 9.$$

Problem 2

Let

$$f(x) = 2x + 3, \quad g(x) = \frac{1}{x}, \quad x \neq 0.$$

(a) (i) Find $(g \circ f)(x)$ and state its domain.

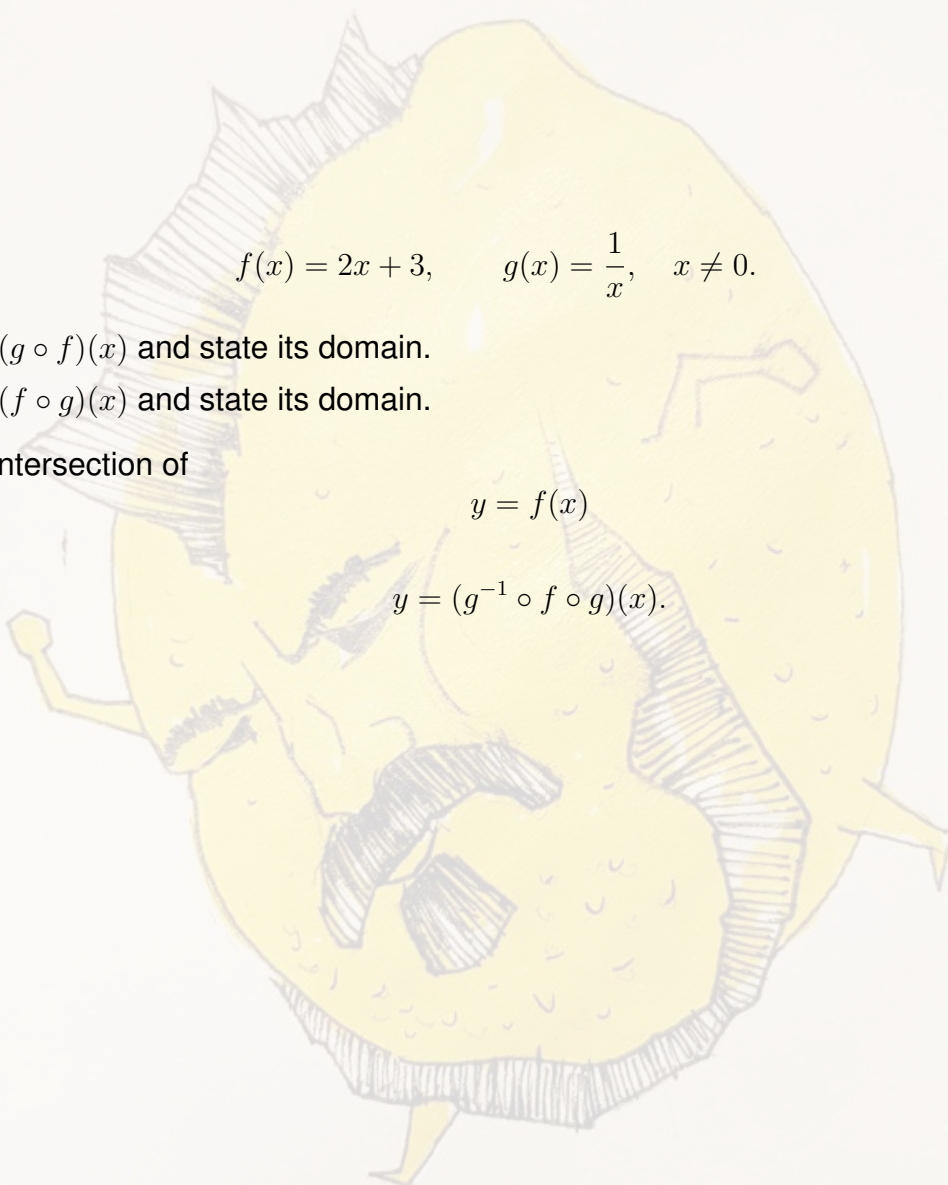
(ii) Find $(f \circ g)(x)$ and state its domain.

(b) Find the intersection of

$$y = f(x)$$

and

$$y = (g^{-1} \circ f \circ g)(x).$$





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