

DP Analysis & Approaches (SL/HL)

Maths with Lemon

Lesson 2.3 Function Basics – Notes

Prior Learning: Number and Algebra	Guidance, clarification and syllabus links
Concept of a function, domain, range and graph.	Function notation such as $f(x)$, $v(t)$ and $C(n)$. The concept of a function as a mathematical model.

1. What is a Function?

A **function** is a relation that assigns **exactly one output** to each input.

one input $\longrightarrow$ exactly one output

Different inputs may have the same output. What is not allowed is one input producing two different outputs.

If $f(x) = 2x + 3$, then

$$f(4) = 2(4) + 3 = 11.$$

Here x is the input (independent variable) and $f(x)$ is the output (dependent variable).

Functions as mathematical models

A function can describe a real situation. For example,

$$C(n) = 5n + 20$$

could model a fixed cost of 20 plus 5 for each item n .

2. Ways to Represent Functions

Functions may be represented by equations, ordered pairs, tables, mapping diagrams and graphs.

Ordered pairs

For

$$\{(3, -5), (2, -5), (-5, -6), (1, -9)\},$$

when $x = 2$ the output is -5 , so

$$f(2) = -5.$$

Tables

If

x	0	1	2	3	4
y	9	18	27	36	45

then

$$f(2) = 27.$$

Equations

If $m(x) = 9x - 2$, then

$$m(2) = 9(2) - 2 = 16.$$

3. Relations and Functions

A **relation** is any collection of input-output pairs. A function is a special type of relation. For

$$\{(2, -5), (2, 7), (-3, -4), (-1, -7)\},$$

the input 2 has two outputs, -5 and 7 . Therefore the relation is **not a function**.

A relation is not a function if at least one input is paired with more than one output.

4. Determining Whether a Relation is a Function

Ordered pairs: inspect the x -coordinates. A repeated x with different y -values means it is not a function.

Mapping diagrams: every input must have exactly one arrow leaving it. Several inputs may point to the same output.

Tables: a repeated input with different outputs means the relation is not a function.

Graphs: Vertical Line Test. A graph represents y as a function of x if every vertical line intersects it at most once. A sideways parabola such as

$$x = (y - 1)^2$$

fails this test because some x -values correspond to two y -values.

5. Domain and Range

The **domain** is the set of all possible inputs. The **range** is the set of all possible outputs.

Domain: x -values

Range: y -values

For

$$\{(2, 3), (5, 7), (2, 6), (-4, 8), (3, 8)\},$$

$$\text{Domain} = \{-4, 2, 3, 5\},$$

$$\text{Range} = \{3, 6, 7, 8\}.$$

Repeated values are listed only once.

6. Domain and Range from Graphs

To find the domain, inspect how far the graph extends **horizontally**. To find the range, inspect how far it extends **vertically**.

- Closed point: endpoint is included, so use $\leq$ or $\geq$.
- Open point: endpoint is excluded, so use $<$ or $>$.
- Arrow: the graph continues indefinitely.

7. Domain Restrictions from Equations

Square roots

For real-valued functions, the expression under a square root must satisfy

$$\text{radicand} \geq 0.$$

For $f(x) = \sqrt{2 - x}$,

$$2 - x \geq 0 \Rightarrow x \leq 2.$$

Also $f(x) \geq 0$, so the range is $[0, \infty)$.

If a square root is in a denominator, it cannot equal zero. For

$$h(x) = \frac{1}{\sqrt{x + 4}},$$

we require $x + 4 > 0$, hence

$$x > -4.$$

Rational functions

A denominator cannot be zero. For

$$g(x) = \frac{x-3}{2x-4},$$

$$2x-4 \neq 0 \Rightarrow \boxed{x \neq 2}.$$

For

$$f(x) = \frac{x}{x^2+1},$$

the denominator is never zero for real x , so the domain is $\mathbb{R}$.

Explicit restrictions

For

$$g(x) = -4x^2 - 2x + 1, \quad x \geq 0,$$

the restriction is part of the definition, so the domain is

$$\boxed{[0, \infty)}.$$

8. Useful Examples

For

$$y = -3(x-5)^2 + 2,$$

the vertex is $(5, 2)$ and the parabola opens downward. Therefore

$$\boxed{\text{Domain} = \mathbb{R}, \quad \text{Range} = (-\infty, 2]}.$$

For

$$y = 2|x+3| - 3,$$

the vertex is $(-3, -3)$, so

$$\boxed{\text{Domain} = \mathbb{R}, \quad \text{Range} = [-3, \infty)}.$$

For

$$x = -2(y-1)^2 - 3,$$

the graph is a sideways parabola. It is a relation, but y is not a function of x .

9. Checklist

1. What is the input and what is the output?
2. Does every input have exactly one output?
3. What are the domain and range?
4. Does a graph pass the vertical line test?
5. Are there square roots, denominators, endpoints, arrows or stated restrictions?



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Mathematics made clear.