

DP Analysis & Approaches SL / HL

Maths with Lemon

Lesson 9.2 Integration by Substitution

Mathematician: _____

SL 5.10	Guidance, clarification and syllabus links
The composites of any of these with the linear function $ax + b$.	Example: $f'(x) = \cos(2x + 3) \Rightarrow f(x) = \frac{1}{2} \sin(2x + 3) + C$
Integration by inspection (reverse chain rule) or by substitution for expressions of the form $\int kg'(x)f(g(x)) dx$	Examples: $\int 2x(x^2 + 1)^4 dx, \quad \int 4x \sin(x^2) dx, \quad \int \frac{\sin x}{\cos x} dx$

Warm Up

Find the indefinite integral of each of the following.

a) $f(x) = 5x^7$	b) $g(x) = 8x^{3/2} - 2 + \frac{1}{x^2}$	c) $h(x) = \frac{1}{x^3} + \frac{1}{x}$
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Integration by Substitution (Reverse Chain Rule)

The chain rule gives

$$\frac{d}{dx} f(g(x)) = f'(g(x))g'(x).$$

Working backwards,

$$\int f'(g(x))g'(x) dx = f(g(x)) + C.$$

Example 1

$$\int (5x + 4)^5 dx$$

Step 1: Set the inner function as u .

Step 2: Find the derivative of u .

Step 3: Solve for dx .

Step 4: Substitute in terms of u and du .

Step 5: Integrate with respect to u .

Step 6: Substitute back in terms of x .

Example 2

$$\int (3x + 2)^4 dx$$

Example 3

$$\int 3t^2(t^3 + 4)^5 dt$$

Example 4

$$\int \sqrt{4x - 5} dx$$

Example 5

$$\int t^2(t^3 + 4)^{-1/2} dt$$

Example 6

$$\int \cos(2x + 1) dx$$

Example 7

$$\int \sin^{10} x \cos x \, dx$$

Example 8

$$\int \frac{(\sqrt{x} - 1)^2}{\sqrt{x}} \, dx$$

Example 9

$$\int \sqrt{x^3 + x^2} (3x^2 + 2x) \, dx$$

Example 10

$$\int e^{-x/\pi + 1/3} \, dx$$

Example 11

Given that

$$\frac{dy}{dx} = x \sin(3x^2),$$

find an expression for y in terms of x .

Example 12

Find

$$\int \left(\frac{e^{2x} - e^{-2x}}{e^x} \right) dx.$$

Example 13

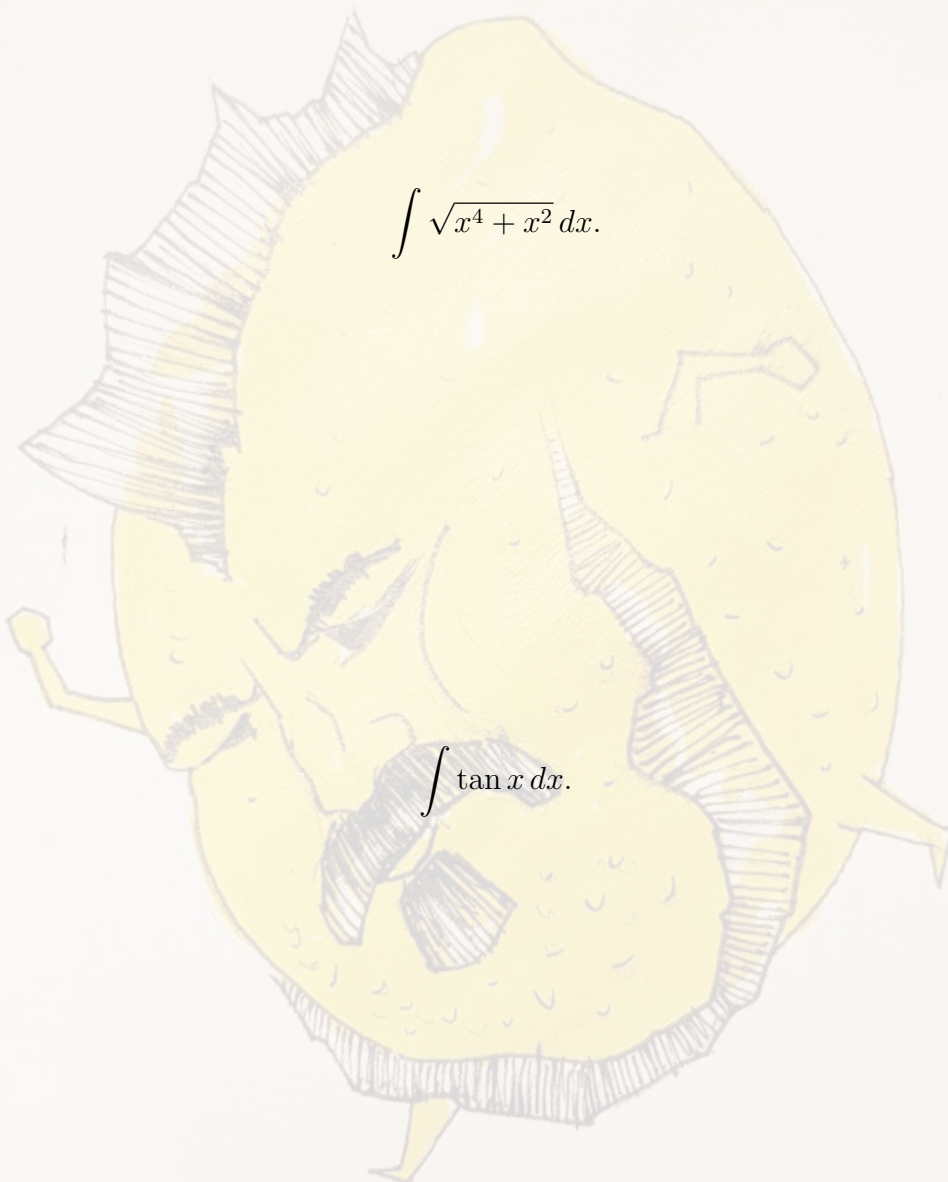
Find

$$\int \sqrt{x^4 + x^2} dx.$$

Example 14

Find

$$\int \tan x dx.$$





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Mathematics made clear.