

# DP Analysis & Approaches HL

Maths with Lemon

## Lesson 9.2HL: Integration by Parts

Mathematician: \_\_\_\_\_

### Warm Up

Use the substitution  $u = \sin x$  to find an expression for

$$\int \frac{3 \cos x}{10 - \cos^2 x} dx.$$

*Hint: use  $\cos^2 x = 1 - \sin^2 x$ .*

### Integration by Parts

$$\int u dv = uv - \int v du$$

For each example, clearly identify  $u$ ,  $dv$ ,  $du$  and  $v$ .

#### Example 1

Find

$$\int x \sin x dx.$$

## Example 2

Use integration by parts to find

$$\int x^3 \ln x \, dx.$$

## Example 3

Find

$$\int x^2 e^x \, dx.$$



## Example 4

Find

$$\int e^x \sin x \, dx.$$

## Example 5

Find

$$\int e^{3x} \sin 2x \, dx.$$



## Extra Practice

(1a)  $\int x \cos 2x \, dx$

(1b)  $\int x \cos 3x \, dx$

(2a)  $\int x \sin \left( \frac{x}{2} \right) \, dx$

(2b)  $\int x \sin \left( \frac{x}{3} \right) \, dx$

(3a)  $\int x e^{-2x} \, dx$

(3b)  $\int x e^{-3x} \, dx$

(4a)  $\int x \ln x \, dx$

(4b)  $\int x^2 \ln x \, dx$

(5a)  $\int \frac{1}{x^2} \ln x \, dx$

(5b)  $\int \frac{1}{x^3} \ln x \, dx$

(6a)  $\int \sqrt{x} \ln x \, dx$

(6b)  $\int \frac{1}{\sqrt{x}} \ln x \, dx$



$$(7a) \int x^2 e^{3x} dx$$

$$(7b) \int x^2 e^{-2x} dx$$

$$(8a) \int x^2 \sin 2x dx$$

$$(8b) \int x^2 \sin 3x dx$$

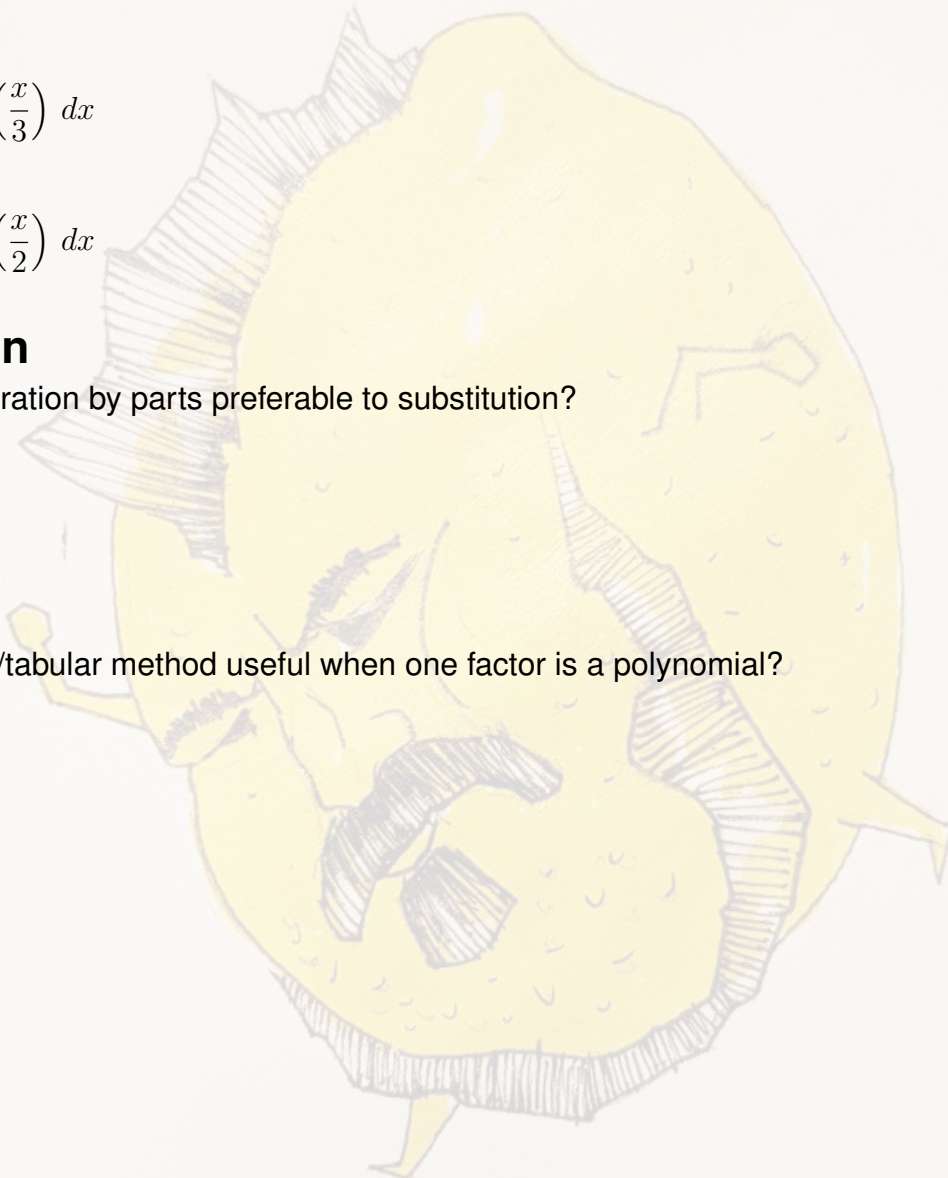
$$(9a) \int x^2 \cos\left(\frac{x}{3}\right) dx$$

$$(9b) \int x^2 \cos\left(\frac{x}{2}\right) dx$$

## Reflection

When is integration by parts preferable to substitution?

Why is the DI/tabular method useful when one factor is a polynomial?





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Mathematics made clear.