

DP Analysis & Approaches HL

Maths with Lemon

Lesson 9.2HL: Integration by Parts — Theory Notes

HL 5.16	Guidance
Integration by parts	Examples: $\int x \sin x \, dx$, $\int \ln x \, dx$, $\int \arcsin x \, dx$.
Repeated integration by parts	Examples: $\int x^2 e^x \, dx$, $\int e^x \sin x \, dx$.

1. Main Formula

From the product rule,

$$\frac{d}{dx}(uv) = u \frac{dv}{dx} + v \frac{du}{dx}.$$

Rearranging after integration gives

$$\int u \, dv = uv - \int v \, du.$$

Integration by parts is therefore the reverse of the product rule.

2. Choosing u and dv

A useful guide is **LIATE**:

Logarithmic $\rightarrow$ Inverse trig $\rightarrow$ Algebraic $\rightarrow$ Trig $\rightarrow$ Exponential.

Usually choose u from the earliest suitable category and let the remaining factor be dv .

3. Example: $\int x \sin x \, dx$

Choose

$$u = x, \quad dv = \sin x \, dx, \quad du = dx, \quad v = -\cos x.$$

Then

$$\int x \sin x \, dx = -x \cos x + \int \cos x \, dx,$$

so

$$-x \cos x + \sin x + C.$$

4. A Single Function

For $\int \ln x \, dx$, write $\int 1 \cdot \ln x \, dx$ and choose

$$u = \ln x, \quad dv = dx.$$

Thus $du = \frac{1}{x} dx$ and $v = x$, giving

$$\int \ln x \, dx = x \ln x - x + C.$$

5. Tabular / DI Method

The uploaded lesson uses the DI method for repeated integration by parts. Differentiate one factor repeatedly, integrate the other repeatedly, and use alternating signs $+, -, +, -, \dots$

For

$$\int x^2 e^x \, dx,$$

sign	D	I
+	x^2	e^x
-	$2x$	e^x
+	2	e^x
-	0	e^x

so

$$\int x^2 e^x dx = x^2 e^x - 2x e^x + 2e^x + C.$$

6. When the Original Integral Reappears

Let

$$I = \int e^x \sin x dx.$$

After integration by parts twice,

$$I = e^x \sin x - e^x \cos x - I.$$

Therefore

$$2I = e^x (\sin x - \cos x)$$

and

$$I = \frac{e^x}{2} (\sin x - \cos x) + C.$$

7. Example: $\int e^{3x} \sin 2x dx$

Repeated integration by parts gives

$$\int e^{3x} \sin 2x dx = \frac{e^{3x}}{13} (3 \sin 2x - 2 \cos 2x) + C.$$

8. Common Mistakes

- Forgetting the minus sign in the formula.
- Choosing u so differentiation makes it more complicated.
- Forgetting to integrate dv to obtain v .
- Missing constants when integrating $\sin(ax)$, $\cos(ax)$ or e^{ax} .
- Stopping when another integration by parts is still required.
- Failing to solve algebraically when the original integral reappears.
- Forgetting $+C$.

9. Quick Summary

$$\int u dv = uv - \int v du$$

Use LIATE as a guide. Use DI/tabular integration when repeated differentiation simplifies one factor. If the original integral returns, collect it on one side and solve.



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Mathematics made clear.